St George's School
Further Mathematics
KS5 Curriculum

PREREQUISITE KNOWLEDGE & SKILLS *The foundations needed to thrive in this subject.*	**Who should study this subject?** Students who are studying A Level Mathematics who have a real flair and passion for the subject and wish to study it in greater breadth and depth. Studying Further Mathematics is particularly beneficial for those who wish to read Mathematics and/or Engineering at university. **Key Skills developed during KS4:** As per A Level Mathematics **St George's course entry requirements:** In addition to the entry requirement for sixth form, a **grade 7** or above in Mathematics. Please note that Mathematics and Further Mathematics can only be taken **together** with **2 other** A Level courses.
QUALIFICATION *Exam Board, aims and objectives.*	**A Level Further Mathematics Edexcel** https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/specification-and-sample-assesment/a-level-l3-further-mathematics-specification.pdf
ASSESSMENT *Internal monitoring and final assessment.*	**Internal Assessment:** ● Internal checkpoints are taken at regular intervals, 2 or 3 topics at a time. ● January Year 13: Mock examinations **Final assessment:** 4 x 90 minute examinations taken at the end of the course: ● Mandatory Core Pure (50%): Core Pure 1 & Core Pure 2 ● Two Options (25% each): Decision Mathematics, Mechanics or Statistics ○ *Students will begin Year 13 studying all three options in order to best ensure the correct two are selected for further study. This decision will be made sometime in the Autumn Term.*
ENRICHMENT *Trips & Visits, wider reading, etc.*	**Visits and Events:** ● Senior Maths Challenge ● Maths Team Challenge ● University of Hertfordshire Problem Solving Workshops ● Mathsfest **Wider reading:** ● Books by Marcus du Sautoy, Rob Eastaway, Hannah Fry, Simon Singh and Ian Stewart ● Podcasts including More or Less, Infinite Monkey Cage, Curious Cases of Rutherford and Fry There are various websites which are useful for wider reading: ● http://www.cut-the-rope.org/ ● https://plus.maths.org/content/
NEXT STEPS *Where this subject can take you.*	**Related University Courses:** As per A Level Mathematics. In addition, students following degree courses in Mathematics and/or Engineering would find Further Mathematics particularly useful. **Career Paths:** As per A Level Mathematics.

Students complete the A Level Mathematics course in one year, gaining the qualification at the end of Year 12. Please see A Level Mathematics for course content. Students then start the Further Mathematics course at the end of Year 12.

	Year 12
Autumn Term	Students will follow the Year 12 Mathematics course
Spring Term	Students will follow the Year 13 Mathematics course
Summer Term	**Topics:** **Core Pure** Complex numbers, Argand diagrams, Roots of polynomials, Series, Volumes of revolution, Matrices, Linear transformations. **Skills:** Understand and use the definitions of imaginary and complex numbers Add and subtract complex numbers Multiply complex numbers Understand the definition of a complex conjugate Divide complex numbers Solve quadratic equations that have complex roots Solve cubic or quartic equations that have complex roots Show complex numbers on an Argand diagram Find the modulus and argument of a complex number Write a complex number in modulus-argument form Represent loci on an Argand diagram Represent regions on an Argand diagram Derive and use the relationships between the roots of a quadratic equation Derive and use the relationships between the roots of a cubic equation Derive and use the relationships between the roots of a quartic equation Evaluate expressions relating to the roots of polynomials Find the equation of a polynomial whose roots are a linear transformation of the roots of a given polynomial Use standard results for $\sum_{r=1}^{n} 1$ and $\sum_{r=1}^{n} r$ Use standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ Evaluate and simplify series of the form $\sum_{r=m}^{n} f(r)$ where $f(r)$ is linear, quadratic or cubic Find the volume of revolution when a curve is rotated around the x axis Find the volume of revolution when a curve is rotated around the y axis Find more complicated volumes of revolution Model real-life objects using volumes of revolution Understand the concept of a matrix Define the zero and identity matrix Add and subtract matrices Multiply a matrix by a scalar Multiply matrices Calculate the determinant of a matrix Find the inverse of a matrix Use matrices to solve systems of equations Interpret simultaneous equations geometrically Understand the properties of linear transformations and represent them using matrices Perform reflections and rotations using matrices Carry out enlargements and stretches using matrices

	Find the coordinates of invariant points and the equations of invariant lines Carry out successive transformations using matrix products Understand linear transformations in 3 dimensions Use inverse matrices to reverse linear transformations **Assessment:** Work will be submitted regularly to assess understanding Checkpoints are taken at regular intervals, 2 or 3 topics at a time

	Year 13
Autumn Term	**Topics:** **Core Pure** Proof by induction, vectors, Complex numbers, Methods in calculus, Series, Volumes of revolution, Methods in differential equations. **Decision (if taken as an option)** Algorithms, Graphs and networks, Algorithms on graphs, Route inspection, The travelling salesman problem **Mechanics (if taken as an option)** Momentum and impulse, Work, energy and power, Elastic strings and springs **Statistics (if taken as an option)** Discrete random variables, Poisson distributions, Geometric and negative binomial distributions, Hypothesis testing, Central limit theorem **Skills:** **Core Pure** Understand the principle of proof by induction and prove results about sums of series Prove results about divisibility using induction Prove results about matrices using induction Understand and use the vector and Cartesian forms of the equation of a straight line in three dimensions Understand and use the vector and Cartesian forms of the equation of a plane Calculate the scalar product for two 3D vectors Calculate the angle between two vectors, two lines, a line and a plane, or two planes Understand and use the scalar product form of the equation of a plane Determine whether two lines meet and determine the point of intersection Calculate the perpendicular distance between: two line, a point and a line, or a point and a plane Express a complex number in exponential form Multiply and divide complex numbers in exponential form Understand de Moivre's theorem Use de Moivre's theorem to derive trigonometric identities Use de Moivre's theorem to find sums of series Know how to solve completely equations of the form $z^n - a - ib = 0$, giving special attention to cases where a = 1 and b = 0 Use complex roots of unity to solve geometric problems Evaluate improper integrals Understand and evaluate the mean value of a function Integrate rational functions using trigonometric substitutions Integrate using partial fractions Understand and use the method of differences to sum finite series Find and use higher derivatives of functions Know how to express functions as an infinite series in ascending powers using Maclaurin series expansion Be able to find the series expansions of compound functions Find volumes of revolution around the x-axis

Find volumes of revolution around the y-axis
Find the volumes of revolution for curves defined parametrically
Model real-life applications of volumes of revolution
Solve first-order differential equations using an integrating factor
Solve second-order homogeneous differential equations using the auxiliary equation
Solve second-order non-homogeneous differential equations using the complimentary function and the particular integral
Find particular solutions to differential equations using given boundary conditions

Decision (if taken as an option)
Use and understand an algorithm given in words
Understand how flow charts can be used to describe algorithms
Carry out a bubble sort
Carry out a quick sort
Carry out the three bin-packing algorithms and understand their strengths and weaknesses
Determine the order of an algorithm
Know how graphs and networks can be used to create mathematical models
Be familiar with basic terminology used in graph theory
Know some special types of graph
Understand how graphs and networks can be represented using matrices
Use the planarity algorithm to determine whether or not a given graph is planar
Use Kruskal's algorithm to find a minimum spanning tree
Use Prim's algorithm on a network to find a minimum spanning tree
Apply Prim's algorithm to a distance matrix
Use Dijkstra's algorithm to find the shortest path between two vertices in a network
Use Floyd's algorithm
Use the orders of nodes to determine whether a graph is Eulerian, semi-Eulerian or neither
Use the route inspection (Chinese postman) algorithm to find the shortest route in a network
Use the route inspection algorithm in networks with more than four odd nodes
Explain the differences between the classical and practical problems
Use a minimum spanning tree method to find an upper bound
Use a minimum spanning tree method to find a lower bound
Use the nearest neighbour algorithm to find an upper bound

Mechanics (if taken as an option)
Calculate the momentum of a particle and the impulse of a force
Solve problems involving collisions using the principles of conservation of momentum
Use the impulse-momentum principle and the principle of conservation of momentum in vector form
Calculate the work done by a force when its point of application moves
Calculate the kinetic energy of a moving particle and the potential energy of a particle
Use the principle of conservation of mechanical energy and the work-energy principle
Calculate the power developed by an engine
Use Hooke's law to solve equilibrium problems involving elastic strings or springs
Use Hooke's law to solve dynamics problems involving elastic strings or springs
Find the energy stored in an elastic string or spring
Solve problems involving elastic energy using the principle of conservation of mechanical energy and the work-energy principle

Statistics (if taken as an option)
Find the expected value of a discrete random variable X
Find the expected value of X^2
Find the variance of a discrete random variable
Use the expected value and variance of a function of X
Solve problems involving random variables
Use the Poisson distribution to model real-world situations
Use the additive property of the Poisson distribution
Understand and use the mean and variance of the Poisson distribution
Understand and use the mean and variance of the binomial distribution
Use the Poisson distribution as an approximation to the binomial distribution
Understand and use the geometric distribution
Calculate and use the mean and variance of the geometric distribution
Understand and use the negative binomial distribution
Calculate and use the mean and variance of the negative binomial distribution

	Use hypothesis tests to test for the mean λ of a Poisson distribution Find critical regions of a Poisson distribution using tables Use hypothesis tests to test for the parameter *p* in a geometric distribution Find critical regions of a geometrical distribution Understand and apply the central limit theorem to approximate the sample mean of a random variable, $\overline{X}$ Apply the central limit theorem to other distributions **Assessment:** Work will be submitted regularly to assess understanding Checkpoints are taken at regular intervals, 2 or 3 topics at a time
Spring Term	**Topics:** **Core Pure** Polar coordinates, Hyperbolic functions, Modelling with differential equations **Decision (if taken as an option)** Linear programming, The simplex algorithm, Critical path analysis **Mechanics (if taken as an option)** Elastic collisions in one dimension, Elastic collisions in two dimensions **Statistics (if taken as an option)** Chi-squared tests, Probability generating functions, Quality of tests **Skills:** **Core Pure** Understand and use polar coordinates Convert between polar and Cartesian coordinates Sketch curves with *r* given as a function of θ Find the area enclosed by a polar curve Find tangents parallel to, or at right angles to, the initial line Understand the definitions of hyperbolic functions Sketch the graphs of hyperbolic functions Understand and use the inverse hyperbolic functions Prove identities and solve equations using hyperbolic functions Differentiate and integrate hyperbolic functions Model real-life situations with first-order differential equations Use differential equations to model simple harmonic motion Model damped and forced oscillations using differential equations Model real-life situations using coupled first-order differential equations **Decision (if taken as an option)** Formulate a problem as a linear programming problem Illustrate a two-variable linear programming problem graphically Locate the optimal point in a feasible region using the objective line (ruler) method Use the vertex testing method to locate the optimal point Determine solutions that need integer values

	Understand and use slack and surplus variables Solve maximising and minimising linear programming problems using simplex tableaux Use the simplex tableau method to solve linear programming problems requiring integer solutions Understand and use the two-stage simplex method for maximising and minimising problems which may include $\leq$ and $\geq$ constraints Understand and use the Big-M method for maximising and minimising problems which may include $\leq$ and $\geq$ constraints Model a project by an activity network using precedence table Use dummy activities Identify and calculate early and late event times in activity networks Identify critical activities Calculate the total float of an activity Calculate and use Gantt (cascade) charts Construct resource histograms Construct scheduling diagrams **Mechanics (if taken as an option)** Solve problems involving the direct impact of two particles by using the principle of conservation of momentum and Newton's law of restitution Apply Newton's law of restitution to problems involving the direct collision of a particle with a smooth plane surface Find the change in energy due to an impact or the application of an impulse Solve problems involving successive direct impacts Solve problems involving the oblique impact of a smooth sphere with a fixed surface Solve problems involving the oblique impact of two smooth spheres Solve problems involving successive oblique impacts of a sphere with smooth plane surfaces **Statistics (if taken as an option)** Form hypotheses about how well a distribution fits as a model for an observed frequency distribution and measure goodness of fit of a model to observed data Understand degrees of freedom and use the chi-squared (χ^2) family of distributions Be able to test a hypothesis Apply goodness-of-fit tests to discrete data Use contingency tables Apply goodness-of-fit tests to geometric distributions Understand the use of probability generating functions Use probability generating functions for standard distributions Use probability generating functions to find the mean and variance of a distribution Know the probability generating function of the sum of independent random variables Know about Type I and Type II errors Find Type I and Type II errors using the normal distribution Calculate the size and power of a test Draw a graph of the power function for a test **Assessment:** Work will be submitted regularly to assess understanding Checkpoints are taken at regular intervals, 2 or 3 topics at a time January: Mock examinations
Summer Term	The summer term is used to consolidate all the knowledge and skills acquired in the course and prepare students for the final examinations.